# Computer lab 4

**Instructions**

* Create a report to the lab solutions in PDF.
* Be concise and do not include unnecessary printouts and figures produced by the software and not required in the assignments.
* **Include all your codes as an appendix into your report.**
* A typical lab report should 2-4 pages of text plus some amount of figures plus appendix with codes.
* The lab report should be submitted via LISAM before the deadline.

***Assignment 1: Computations with Metropolis-Hastings***

Consider the following probability density function:

You can see that the distribution is known up to some constant of proportionality.

1. Use Metropolis-Hastings algorithm to generate samples from this distribution by using proposal distribution as log-normal LN(*Xt*, 1), take some starting point. Plot the chain you obtained as a time series plot. What can you guess about the convergence of the chain? If there is a burn-in period, what can be the size of this period?
2. Perform step 1 by using chi-square distribution as proposal distribution where floor(x) means integer part of *x*.
3. Compare the results of steps 1 and 2 and make conclusions.
4. Generate 10 MCMC sequences using the generator from the step 2 and with starting points 1,2,.., or 10. Use Gelman-Rubin method to analyze convergence of these sequences.
5. Estimate using the samples from steps 1 and 2.
6. The distribution generated is in fact a gamma distribution. Look in the literature and define the actual value of the integral. Compare it with the one you obtained.

***Assignment 2: Gibbs sampling***

A concentration of a certain chemical was measured in a water sample, and the result was stored in the data **chemical.RData** having the following variables:

* X: day of the measurement
* Y: measured concentration of the chemical.

The instrument used to measure the concentration had certain accuracy; this is why the measurements can be treated as noisy. Your purpose is to restore the expected concentration values.

1. Import the data to R and plot the dependence of Y on X. What kind of model is reasonable to use here?
2. A researcher has decided to use the following (random-walk) Bayesian model (*n*=number of observations, are unknown parameters):

where the prior is

Present the formulas showing the likelihood and the prior (hint: a chain rule can be used here )

1. Use the Bayes theorem to get the posterior up to a constant of proportionality, and then find out the distributions for where is a vector containing all values except of
   1. Hint A: consider separate formulas for and then a formula for all remaining
   2. Hint B:
   3. Hint C:
2. Use the distributions derived in step 3 to implement a Gibbs sampler that uses as a starting point. Run the Gibbs sampler to obtain 1000 values of  and then compute the expected value of  by using Monte Carlo approach. Plot the expected value of  versus X and Y versus X in the same graph. Does it seem that you have managed to remove the noise? Does it seem that the expected value of  can catch the true underlying dependence between Y and X?
3. Make a trace plot for and comment on the burn-in period and convergence.

***Submission procedure***

**Assume that X is the current lab number.**

**If you are neither speaker nor opponent for this lab,**

* Submit your report using *Lab X* item in the *Submissions* folder before the deadline.
* Make sure that you or some of your group members submits the group report using *Lab X group report*  in the *Submissions* folder before the deadline

**If you are a speaker for this lab,**

* Submit your report using *Lab X* item in the *Submissions* folder before the deadline.
* Make sure that you or some of your group members does the following before the deadline:
  + submits the group report using *Lab X group report*  in the *Submissions* folder before the deadline
  + Goes to Study room *Speakers X🡪Documents* and opens file *Password X.txt*. Then the student should put your group report into ZIP file *Lab X.zip* and protect it with a password you found in Password X.txt
  + Uploads the file to *Collaborative workspace* folder

**If you are opponent for this lab,**

* Submit your report using *Lab X* item in the *Submissions* folder before the deadline.
* Make sure that you or some of your group members submits the group report using *Lab X group report*  in the *Submissions* folder before the deadline
* After the deadline for the lab has passed, go to Collaborative workspace folder and download *Lab X.zip*. Open the PDF in this ZIP file by using the password available in *Course Documents🡪Password X.txt,* read it carefully and **prepare at least two questions/comments/improvement suggestions** in order to put them at the seminar (i.e. at least two questions per opponent)